QUIZ 16 SOLUTIONS: LESSONS 22-23
OCTOBER 27, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [3 pts] Find $\frac{dz}{dt}$ given

$$z = xe^y, \quad x = t^2 + 7t, \quad y = \sin t.$$

(You don't need to simplify, but leave your answer as you would on the homework.)

Solution: Our multivariable chain rule formula is

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}.$$

Differentiating, we get

$$\frac{\partial z}{\partial x} = e^y \quad \frac{\partial z}{\partial y} = xe^y$$

$$\frac{dx}{dt} = 2t + 7 \quad \frac{dy}{dt} = \cos t$$

Hence,

$$\frac{dz}{dt} = \boxed{e^y(2t+7) + xe^y(\cos t)}.$$

2. [7 pts] Find and classify the critical points of

$$f(x, y) = \frac{x^3}{3} - \frac{x^2}{2} - 12x + \frac{y^3}{3} - \frac{y^2}{2} - 2y.$$

Solution: We go through our steps.

Step 1: Find critical points

We get

$$f_x = x^2 - x - 12 \text{ and } f_y = y^2 - y - 2.$$

So,

$$0 = f_x = x^2 - x - 12 = (x-4)(x+3)$$

implies $x = -3, 4$ and

$$0 = f_y = y^2 - y - 2 = (y-2)(y+1)$$

implies $y = -1, 2$. Thus, our critical points are

$$(-3,-1), \quad (-3,2), \quad (4,-1), \quad \text{and} \quad (4,2).$$

Step 2: Find second order derivatives

Differentiating,

$$f_{xx} = 2x - 1, \quad f_{yy} = 2y - 1, \quad f_{xy} = 0.$$

Step 3: Find discriminant

The formula for the discriminant is

$$D = f_{xx}f_{yy} - f_{xy}.$$

Hence,

$$D(x,y) = (2x-1)(2y-1) - (0)^2 = (2x-1)(2y-1).$$

Step 4: Apply test

Critical Points	$D(x_0, y_0)$	$f_{xx}(x_0, y_0)$	Classification
$(-3,-1)$	$\underbrace{(2(-3)-1)}_{<0}\underbrace{(2(-1)-1)}_{<0} > 0$	$2(-3)-1 < 0$	local max
$(-3,2)$	$\underbrace{(2(-3)-1)}_{<0}\underbrace{(2(2)-1)}_{>0} < 0$	$- - -$	saddle point
$(4,-1)$	$\underbrace{(2(4)-1)}_{>0}\underbrace{(2(-1)-1)}_{<0} < 0$	$- - -$	saddle point
$(4,2)$	$\underbrace{(2(4)-1)}_{>0}\underbrace{(2(2)-1)}_{>0} > 0$	$2(4)-1 > 0$	local min